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BOGOLIUBOV SPECTRUM OF INTERACTING BOSE GASES

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Abstract. We study the large-N limit of a system of N bosons interacting with a potential of intensity $1/N$. When the ground state energy is to the first order given by Hartree's theory, we study the next order, predicted by Bogoliubov's theory. We show the convergence of the lower eigenvalues and eigenfunctions towards that of the Bogoliubov Hamiltonian (up to a convenient unitary transform). We also prove the convergence of the free energy when the system is sufficiently trapped. Our results are valid in an abstract setting, our main assumptions being that the Hartree ground state is unique and non-degenerate, and that there is complete Bose-Einstein condensation on this state. Using our method we then treat two applications: atoms with "bosonic" electrons on one hand, and trapped 2D and 3D Coulomb gases on the other hand.

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